Optimizing the design of mechanical components for reliability and cost

Robert Tryon*, Animesh Dey* and Sankaran Mahadevan†

*VEXTEC Corporation, Brentwood, TN, USA.

†Vanderbilt University, Nashville, TN, USA.

Abstract: This paper discusses a design optimization methodology that uses probabilistic methods. The methodology addresses product performance, reliability and cost that allows the engineer to evaluate the impact of design changes on all relevant measures of the design simultaneously. By considering variation in the design parameters, the influence of uncertainty can be determined and the design can be optimized to prevent early failures while minimizing overall cost. The sensitivities of the design variables to the statistical distribution parameters are determined. The sensitivities are then used in an optimization model to determine the best combination of design parameters including the manufacturing tolerances and acceptance testing routines. A demonstration problem that illustrates the application of the method is provided.

Keywords: airfoil design, composite laminate vibration, first order reliability method, manufacturing cost, uncertainty-based optimization.

Reference to this paper should be made as follows: Tryon, R., Dey, A. and Mahadevan, S. (2002) 'Optimizing the design of mechanical components for reliability and cost', *Int. J. of Materials & Product Technology*, Vol. 17, Nos 5/6, pp. 338–352.

1 Introduction

Reliability is the probability that the designed system will function in a satisfactory manner for a pre-specified interval. In confronting reliability, the engineer is concerned with the product reliability at the time of manufacture, as well as the level of reliability exhibited by the product at any time in the future. Initial or zero-time reliability is a function of both the design and the manufacturing process, whereas time-dependent reliability considers the degradation effects on the product. Probabilistic methods are effective tools developed to assist the engineer in predicting the product reliability, in its operating environment as a function of product usage.

Historically, engineering has focused on the development of deterministic predictive methods of product performance. Deterministic methods can only provide single point estimates of the system behavior or response. Consequently, engineers have had to rely on three means of obtaining product assurance:

1. Factors of safety
2. Minimum properties
3. Maximum loads

The inability of deterministic methods to derive the statistical distribution of the system response prevents the direct estimation of the system reliability. Typically, deterministic methods focus on designing the product to sustain a pre-established load. In the prevalent design approach, the product need is identified, conceptual product designs are considered and the design is refined by modifying its load bearing capacity. Since ensuring the load bearing capacity of a design must exceed the in-service environment, operating loads cannot be used to predict product failures. Prototypes are built and tested to extreme conditions, with testing information being used to improve the design configuration. Repetitive evaluation of design improvements, it is hoped, will yield a feasible final product configuration.

Why does designing to load not allow us to predict product failure? Designing to load can only be used to predict product failures if we know with absolute certainty the values of every factor that affects the behavior of the product. Since there is some variability in materials, processes and operating conditions, we do not know the true singular value of any variable that affects a product's performance.

In an increasingly competitive environment, product design forms the foundation for enduring sales responsiveness and enhanced customer satisfaction. Superior product performance, quality and reduced product cost are the results of effective, efficient engineering and design.

The impact of product design on product cost and the profitability of a manufacturing company are significant. Numerous studies have repeatedly demonstrated that over 70 percent of the life cycle cost of a product is determined by its conceptual design (Nevins and Whitney, 1989). Methods that can predict product performance, reliability and cost early in the development process will dramatically enhance corporate profitability and competitiveness.

Despite the overwhelming impact early design decisions have on product cost and performance, present day engineering design methods are unable to determine analytical relationships between factors that affect product performance, cost, and reliability. Evaluation and selection of alternative conceptual designs based on product performance, reliability or cost cannot be accomplished with existing design methods.

The proposed approach determines the relationship between design variables and manufacturing costs. This provides an analytical relationship between variables and cost, allowing engineers to predict final product cost. The cost-design relationship is coupled with probabilistic methods in an overall design optimization algorithm. The integrated cost-reliability relationship is then solved using optimization methods with the results indicating the specific changes required for conceptual designs to attain the cost, performance and reliability goals for the product. This approach permits engineers to study product designs with respect to cost, reliability and performance during the conceptual design phase and enables the integrated method to identify design changes that improve performance and reduce cost.

By employing statistical information on product design variables, the proposed method identifies those variable parameters that significantly effect performance, reliability and cost. More importantly, the method indicates if the average value, variability or inspections are the most important factors affecting performance and cost for each product variable. Knowledge of how controlling significant variables affects product attributes, allows management to focus efforts on controlling significant variable characteristics, while efforts directed at inspecting and controlling non-significant variables can be eliminated.

Probabilistic design optimization requires that parameters being optimized be described by random variables with specific distribution parameters. The challenge in probabilistic design optimization is to develop a formulation of the problem that is suitable for solution. Several alternative approaches have been advanced to undertake optimization under the assumption of uncertainty.

The techniques for handling uncertainty have been employed in the construction of probabilistic reliability optimization methods. Reddy *et al.* (1994) employed a modified safety index optimization approach that used a deterministic term to correct for the higher order error. The authors employed the sensitivity factors developed by Kwak and Lee (1987), which considers the sensitive of the reliability index to the random variable.

Probabilistic reliability-based optimization presents some conflicts where component reliability is optimized but overall system reliability is suboptimal. In this case, a balanced component-system approach has been developed which optimizes component reliability and then verifies if system level reliability requirements are met (Pu *et al.*, 1997).

If the engineer is to control the cost and performance of the product, it is essential to understand the impact of individual statistical distribution parameters on measures of performance and reliability. Once known, the engineer can determine the appropriate level for dimensional and process tolerance that meet cost, performance and reliability goals.

2 First order reliability method (FORM)

The first step in estimating the failure probability is to define a performance function $g(\vec{x})$ corresponding to each failure mode, where $\vec{x}$ is the vector of random variables. The performance function is written in such a way that $g(\vec{x}) > 0$ represents success, $g(\vec{x}) < 0$ represents failure and $g(\vec{x}) = 0$ is referred to as the limit state. For example, in the simple case of an axially loaded bar, if R is the strength, and S is the applied load, then the limit state is written as:

$$g(R, S) = R - S. \tag{1}$$

In this formulation, $g(R, S) < 0$ denotes the failure state, while $g(R, S) > 0$ denotes the safe state. $g(R, S) = 0$ is known as the limit state that separates the failure and the safe regions. The probability of failure for this performance function is

$$p_f = P(R < S) = P\{g(R, S) < 0\} \tag{2}$$

where $P(R < S)$ denotes the probability of occurrence of the event $R < S$. The probability of failure, is now quantitatively obtained as:

$$p_f = \iint_{\Omega} f_{R,S}(r,s)\mathrm{d}r\,\mathrm{d}s \tag{3}$$

where Ω is the failure set, i.e., the set of all values of R and S such that $g(R,S) < 0$, $f_{R,S}(r,s)$ is the joint probability density function of all the random variables. In general, it is difficult to know the joint density function, except when the random variables are independent. It is difficult to develop $f_{R,S}(r,s)$, even for widely used distributions such as normal and lognormal. If the joint density function is available, the above integral is multidimensional and intractable for a realistic problem.

One option is to pursue Monte Carlo simulation. The structure is analyzed a large number of times with sets of randomly generated values of the design parameters. Even with efficient sampling techniques, simulation can be time consuming and expensive.

Alternatively, an elegant first order reliability method (FORM) has been developed to compute an approximate estimate of the failure probability. All the variables ($\vec{x}$) are transformed to equivalent uncorrelated standard normal variables ($\vec{y}$). In the $\vec{y}$ space, a linear approximation is constructed to the limit state at the point of minimum distance from the origin, referred to as the Most Probable Point (MPP) as shown in Figure 1. The search for the minimum distance point is an optimization problem, formulated as:

$$\begin{aligned}&\text{Minimize } D = \sqrt{\vec{y}^T\vec{y}}\\&\text{such that } g(\vec{y}) = 0.\end{aligned} \tag{4}$$

Several optimization routines are available to solve the above-constrained optimization problem. The method used in this paper was formulated by Rackwitz and Fiessler (1974). A first order estimate of the failure probability is then computed as

$$p_f = \phi(-\beta) \tag{5}$$

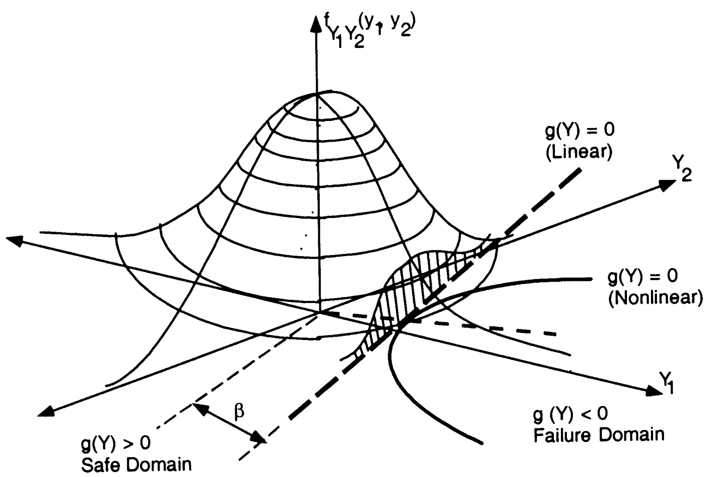

Figure 1 *Graphical representation of the reliability method for two random variables.*

where β is the reliability index representing the distance from the origin to MPP, and $\phi(\bullet)$ is the cumulative distribution function of a standard normal variable (i.e., a normal variable with zero mean value and unit standard deviation).

The main benefit of using probabilistic reliability techniques such as FORM to predict product performance is that they enable the engineer to determine the sensitivity of the probability of failure to individual random variables. Specifically, if probabilistic methods are used, the sensitivity of the probability of failure to changes in random variable mean and variance can be approximated in closed form . Madsen *et al*. (1986) demonstrated the sensitivity of the probability of failure to changes in random variable mean and variance. The authors demonstrated that the sensitivity of the reliability index to changes any single distribution parameter (i.e., mean value or standard deviation) is given by,

$$\begin{aligned}\frac{\partial}{\partial p_i}\beta(p_0) &= \frac{1}{\beta}\vec{y}^{*T}\cdot\frac{\partial}{\partial p_i}\vec{y}^{*} \\ &= \frac{1}{\beta}\vec{y}^{*T}\cdot\frac{\partial}{\partial p_i}T(z^*, p_0)\end{aligned} \tag{6}$$

where p_0 are the statistical moments of random variable, p_i is the i^{th} moment, y^* is the MPP and z^* is the inverse Rosenblatt transformation (Rosenblatt, 1952). Madsen *et al*. (1986) also demonstrated the sensitivity of the probability of failure to changes in the reliability index, given as

$$\frac{\partial}{\partial p_i}P\big(F(\vec{y}, p_0)\big) \approx \phi\big(-\beta(p_0)\big)\cdot\frac{\partial}{\partial p_i}\beta(p_0) \tag{7}$$

where $F(\vec{y}, p_0)$ is the failure domain. Equations (6) and (7) can be combined as

$$\frac{\partial}{\partial p_i}P\big(F(\vec{y}, p_0)\big) \approx \phi\big(-\beta(p_0)\big)\cdot\frac{1}{\beta}\vec{y}^{*T}\cdot\frac{\partial}{\partial p_i}T(z^*, p_0). \tag{8}$$

Equation (8) estimates the sensitivity of the failure probability to changes in the distribution parameters of the design variables. These sensitivities are used to determine the optimum design configuration.

3 The optimization method

Having defined how to determine product performance and reliability, as well as failure sensitivity to random variable distribution parameter changes, we can define an uncertainty based design optimization method (Kowal *et al*., 1998). The methodology indicates the optimal means of improving product reliability while considering physical, performance and cost objectives as well. In order to evaluate the design we require the sensitivity of performance, reliability and cost to changes in random variable distribution parameters.

The sensitivity of the product performance to changes in the mean and variance of random variables can be determined from equation (8) in closed form for several different distribution types. The determination of the cost sensitivities to changes in the random variables is normally dependent on the specific processes and materials used by the manufacturer. The impact of proposed changes on the manufacturing cost of the product must be quantified. It is not necessary to determine the total manufacturing costs of the design, but merely the differential cost associated with specific changes in the design. The determination of such differential costs will be dependent on the specific means used to affect the changes in the design.

Some manufacturing costs will be determined from proprietary data. Other costs can be determined from existing, approximate estimation technique and databases. Several sources exist that highlight the relative costs associated with specific manufacturing processes, process capabilities and materials (Trucks, 1987; Anon, 1980).

With the cost, performance and reliability sensitivities of the design determined, the effect of a set of design changes on product cost can be estimated. The change in the product reliability or performance can be estimated as,

$$\Delta P\left(F\left(\vec{y}^*, p_0\right)\right) \approx \sum_{i=1}^{n}\left(\frac{\partial}{\partial p_i} P\left(F\left(\vec{y}^*, p_0\right)\right) \cdot \Delta p_i\right) \tag{9}$$

where Δp_i is the change in parameter i and n is the number of changes in random variable distribution parameters. The change in product cost can be estimated as,

$$\Delta C(p_0) \approx \sum_{i=1}^{n}\left(\frac{\partial}{\partial p_i} C(p_0) \cdot \Delta p_i\right) \tag{10}$$

where $\frac{\partial}{\partial p_i} C(p_0)$ is the cost sensitivity due to changes in distribution parameter i.

With these relationships, the cost and performance of the product design can be estimated. A constrained optimization problem can be formulated to determine the optimal design changes required of a product. A linear programming model can now be formulated with the objective being either maximization of product performance or minimization of product cost.

Let us examine the case of determining the minimum cost of a product for a specified level of performance. This problem is one of minimizing the changes in product manufacturing cost, subject to a number of constraints. Among the constraints is a required level of product performance, as well as limitations on the allowable changes to each random variable in the design. The linear programming formulation is

$$\text{MIN Cost} = \sum_{i=1}^{n}\left(\frac{\partial}{\partial p_i} C(p_0) \cdot \Delta p_i\right). \tag{11}$$

Subject to

$$\sum_{i=1}^{n}\left(\frac{\partial}{\partial p_i} P\left(F\left(\vec{y}^*, p_0\right)\right) \cdot \Delta p_i\right) \leq Q$$

$$\forall \Delta p_i \leq L_i$$

$$\forall \Delta p_i \geq 0$$

where Q is the required change in the probability of failure and L_i is the maximum allowable change in distribution parameter i.

Although product cost is considered as the objective function, other criteria can be employed. Interchanging the constraints with the objective function would enable determining the maximum performance attainable for a given cost. The remaining constrains on the feasible changes to random variables would be unchanged.

Having formulated the linear programming problem of interest, the feasible solution to the optimized design changes can be determined. The feasible design changes indicate the optimal changes in random variables that are required to improve the objective function. However, the indicated design changes are the specific indications for improving design from the current design configuration. This implies that at successive design changes new estimates of the probability of performance using probabilistic methods as well as new random variable changes must be generated. The result is an iterative process of successive estimations of performance, optimal design changes, and re-evaluation of designs until a satisfactory, improved design is determined.

The procedure to undertake the integrated design method can be summarized in the following steps:

Step 1: Define the initial product design configuration.

Step 2: Define the appropriate limit states that are likely to affect the proposed design.

Step 3: Identify the feasible changes in the distribution parameters of every variable in the limit state function.

Step 4: Use FORM to estimate the probability of failure or performance, $P(F)$, and the most probable point, y^* for each probabilistic constraint. If all meet their performance specifications, stop; otherwise continue to step 5.

Step 5: Calculate the probability of failure or performance sensitivities, $\partial P(F)/\partial p_i$, for the feasible changes in distribution parameters of every random variable for each constraint.

Step 6: Calculate the cost sensitivities, $\partial C/\partial p_i$, for the feasible changes in distribution parameters of every random variable.

Step 7: Formulate the linear programming model required.

Step 8: Use the linear program to determine the required changes in primitive variable distribution parameters, Δp_i, return to step 4 and update the estimated product probability of failure.

A flowchart outlining this process is shown in Figure 2.

4 Demonstration problem

The uncertainty-based optimization methodology will be demonstrated using a vibration control problem. Consider a cantilever flat plate airfoil that is excited by an aerodynamic loading. The plate is a fiber reinforced composite laminate as shown in Figure 3. The plate is considered to fail when the natural modal frequencies of the plate coincide with the vibration frequencies of the aerodynamic loading. The problem is to find the optimum plate design to minimize the probability of frequency coincidence.

Although the demonstration problem is simple, it can form the basis for several

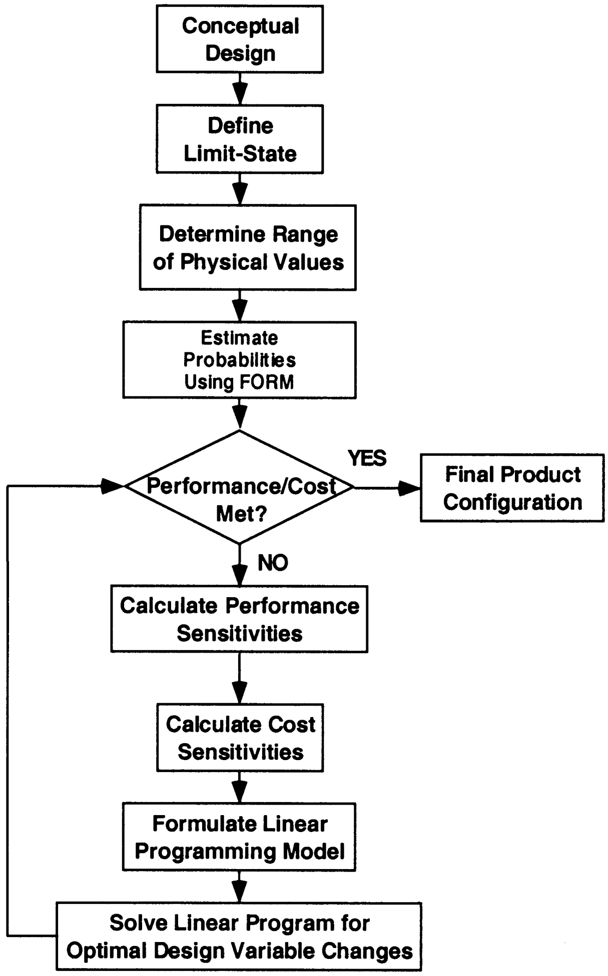

Figure 2 *Sequential linear optimization model.*

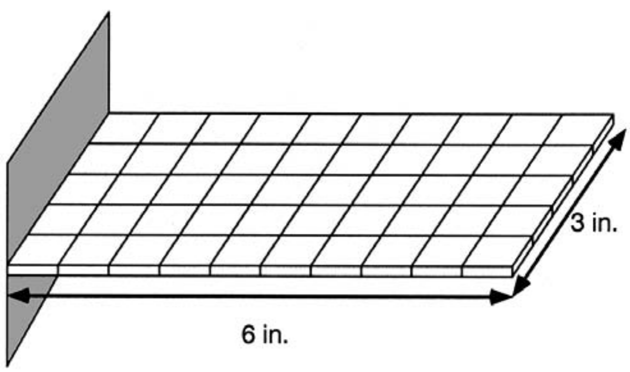

Figure 3 *Finite element model of flat cantilever plate airfoil.*

important classes of problems that involve tailoring for structural control without changing airfoil shape. Applications include stall and noise control of fixed wing airfoils, noise and flutter control of rotor wing airfoils and high cycle fatigue control of turbine blades and vanes. Changes in the airfoil design to avoid vibration induced failures must be made within the confines of acceptable aerodynamics. The non-vibratory structural integrity must also be maintained.

The applied loading considered in this paper is narrow band. The narrow band loading simulates the harmonic flow disturbances inherent in rotating machinery. Narrow band loading is applicable to the propeller wash over a fixed wing, blade-fuselage interference for rotor wings, and blade-vane interference and rotating stall for gas turbine airfoils. The current study considers the frequency content of a narrow band loading caused by a fix structural interference. The narrow band loading is harmonic or near harmonic in nature and is characterized by a high amount of energy over a narrow range of frequencies. Due to the width of the frequency band, resonance caused by the coincidence of response frequencies with input frequencies can be avoided through judicial structural tailoring. The excitation frequency of a fix structural interference is usually a direct linear function of the engine speed. The excitation frequencies to be avoided occur at critical engine speeds such as full power take-off and flight idle.

For the demonstration purposes in the current paper, two arbitrary critical excitation frequencies will be consider, a flight idle excitation of 250 Hz. and a full power take-off of 500 Hz. The demonstration problem will consider optimum plate design to avoid resonance while maintaining overall stiffness.

Natural vibratory frequencies of solid airfoils are directly proportional to the thickness to chord ratio, the thickness to span ratio and the square root of the specific modulus (elastic modulus/density). Most metallic materials used in aerostructure design such as steel, aluminum, titanium and nickel have approximately equal specific modulus and changing the material will not appreciably affect the airfoil frequencies. Therefore, with metallic airfoils, the designer can change the frequencies by changing the geometry. Changes in the airfoil geometry are severely limited by aerodynamic and non-vibratory structural criteria. The geometry changes are usually limited to small changes in airfoil thickness and changes in the compliance at the airfoil fixities such as z-shrouds and platform damper for gas turbine blades. The only other option to change the vibratory characteristics is to change the frequency of the flow disturbance. This usually requires a fundamental change in the entire system. The use of laminate composites adds greatly to the designer's ability of influence the vibratory characteristics of the airfoil. The ply angles and the locations of the plies within the airfoil can be varied to affect specific modes.

A family of graphite/epoxy laminates were investigated to determine the ply angle effect on blade frequencies. A finite element model was constructed of a $6'' \times 3''$ plate (aspect ratio of 2) with an element grid of 10×5 for a total of 50 elements as shown in Figure 3. The elements are 4-noded quadrilateral plate elements for a total of 66 nodes. The elements account for shear deformation using Reissner-Mindlin plate and shell theories. Subspace iteration is used to evaluate the eigenvalues and eigenvectors. Material properties are shown in Table 1. Node line plots of typical mode shapes of a cantilever flat plate with an aspect ration of 2 are shown in Figure 4.

Table 1 Material properties for finite element model.

$E_{11} = 18.5$ Msi	$E_{22} = 1.6$ Msi	$G_{12} = 0.65$ Msi
$\nu_{12} = 0.25$	$\nu_{23} = 0.23$	$\rho = 0.055$ lb/in^3

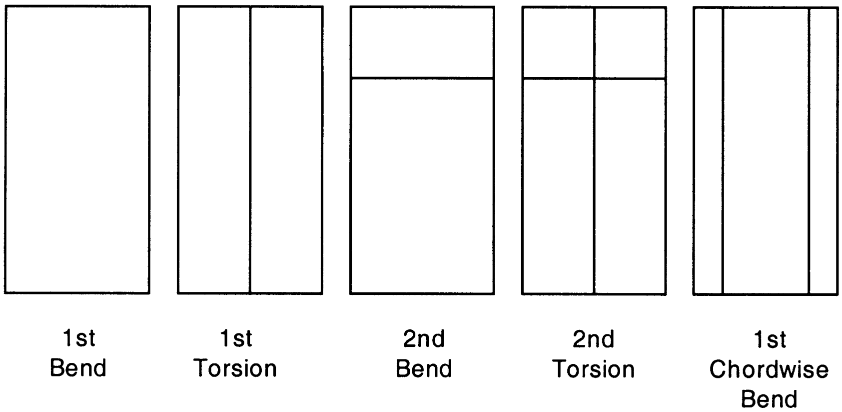

Figure 4 *Typical modes shapes of cantilever flat plates.*

The stacking sequence of the family is $[\pm\alpha/0/90]_S$ in which the ply angles of the outermost plies are varied to determine the ply angle effect on frequency. To determine which ply angles are stiffest in bending, torsion and chord-wise bending, consider Figure 5. The bending modes are at higher frequencies at $\alpha = 0$. This is because the bending modes have maximum strain along the 0° axis. The outermost

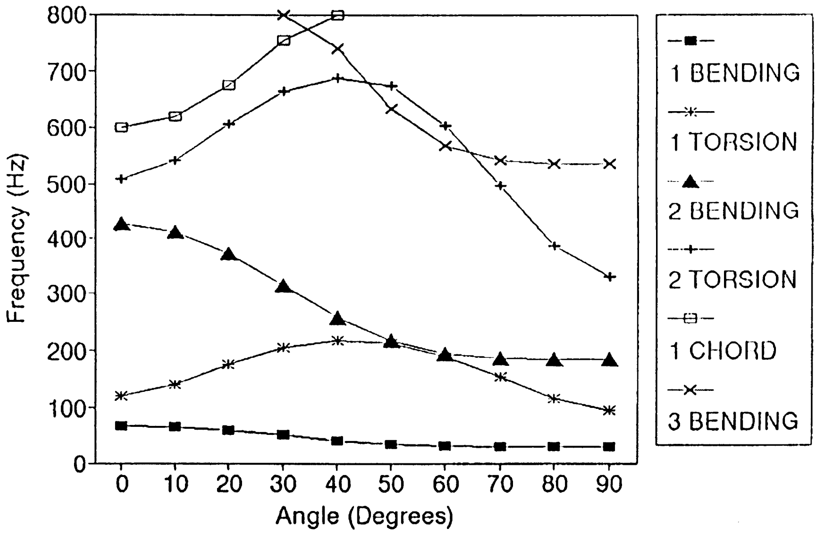

Figure 5 *Frequency vs. ply angle for* $[\pm\alpha/0/90]_S$ *laminate.*

plies are stiffest along this axis. Similarly, the torsional modes are highest at $\alpha = 45^\circ$ because the maximum strain is at 45°. The chord-wise bend mode is highest at $\alpha = 90^\circ$ because the maximum strain is at 90°.

Adjusting the outermost ply angles of the $[\pm\alpha/0/90]_S$ laminate allows for tailoring the blade vibration to provide large frequency range free of modes. For example, if the flow disturbance had excitations between 200 and 300 Hz, alpha could be chosen between 0° and 20°. If the flow disturbance had frequencies between 300 and 600 Hz, alpha could be chosen at 40°. If one particular mode is of interest, that mode could be driven up or down.

As mentioned earlier, the engineer can make geometric adjustments in the design that are usually limited to small changes in airfoil thickness and changes in the compliance at the airfoil fixities such as *z*-shrouds and platform damper for gas turbine blades. Therefore, airfoil fixity compliance and airfoil thickness were also considered as allowable design changes. In the present example, the compliance is modeled by placing a small spring at the fixed end of the cantilever plate. The compliance of the spring has a nominal value of 6.375. The actual design does not have a spring at the fixity, but the spring simulates the compliance of the blade stalk (region of the blade below the platform). The plate thickness has a nominal value of 0.0052 inches.

There is a manufacturing cost associated with the nominal value of the thickness i.e., the thicker the plate the more material used and therefore, an increase in cost. Some design parameters do not have a cost associated with their nominal values. There is no nominal cost associated with the ply angle of the outermost plies. It is assumed that it is no more expensive to manufacture an airfoil with 45 degree plies than an airfoil with 60 degree plies. Likewise, it is assumed that the design of the airfoil stalk (which governs the nominal fixity compliance) does not affect manufacturing cost.

The manufacturing variations are described by the standard deviations of the statistical distributions of the design parameters. The standard deviations are dependent on the manufacturing techniques and the care that must be taken to hold tolerances. In general, manufacturing variations have a large impact on cost i.e., the tighter the tolerance that must be maintained, the higher the cost.

The design objective is to determine the ply angle, plate thickness and compliance required minimizing manufacturing cost and avoiding interference frequencies while restricting the maximum tip displacement of the plate. Seven limit state equations were required to approximate the relationship between the design parameters and each of the six vibrations modes and the tip displacement. Since an explicit mathematical relationship did not exist between the design parameters and the necessary response, the finite element model was used in a virtual design of experiments. For the design of experiments, different sets of input values of the random variables are used to compute the corresponding values of the response. The design of experiment matrix was constructed with the following inputs:

Thickness	0.00364, 0.0052, 0.00676
Compliance	6.0, 6.375, 6.75
Ply angle	0–90 in 10 degree increments

where the thickness is in inches and the compliance is in inches per pound.

The response surface equation for the 2nd bending mode was found to be

$$f = (191.8239t)(39.66861c^{-1.986}) \times (434.085 - 0.155\alpha^2 + 3.584 \times 10^{-5}\alpha^4 - 3.11 \times 10^{-9}\alpha^6 + 8.983 \times 10^{-18}\alpha^{10}) \quad (12)$$

where f is the frequency, t is the thickness, c is the fixity compliance and α is the ply angle. A similar equation was developed for the other five modes. The response surface equation for the tip displacement was found to be

$$D = 7.27 + 6.11 \times 10^{-3}\alpha - 1.46 \times 10^{-5}\alpha^2 - 2.20 \times 10^3 t + 1.67 \times 10^5 t^2 - 0.280c + 4.34 \times 10^{-2}c^2. \quad (13)$$

The eight steps outlined earlier are applied to the demonstration problem.

Step 1: Define the initial plate configuration. All of the variables are assumed to have a Gaussian distribution.

Variable	*Mean*	*Std. Deviation*
Thickness	0.0416	0.00416
Compliance	6.375	1.275
Ply Angle	45.0	3.0

Step 2: The limit state equations for the six mode shape frequencies and the tip displacement (as in Eq. (12) and (13)) are used.

Step 3: The allowable changes in the design parameters are defined.

Variable	*Initial Value*	*Allowable Range*
Thickness μ	0.0416	0.0316–0.1416
Thickness σ	0.00416	0.00216–0.00016
Compliance μ	6.375	5.0–7.50
Compliance σ	1.275	1.0–2.0
Ply Angle μ	45.0	0–90
Ply Angle σ	5.0	2.0–12.5

Step 4: The probability of failure and MPP are found for each vibratory mode at each interference frequency and the tip displacement. There are 13 constraints, two interference frequencies for each mode and the tip displacement. (Only the results for the second bending mode and tip displacement are shown for brevity.)

Constraint	*P.O.F*	*MPP*
2nd Bending, 250 Hz	0.187	$y_t^* = 0.0414$ $y_c^* = 6.581$ $y_\alpha^* = 45.148.$
2nd Bending, 500 Hz	0.0469	$y_t^* = 0.043$ $y_c^* = 4.760$ $y_\alpha^* = 44.922.$
Tip displacement	0.06	$y_t^* = 0.043$ $y_c^* = 5.912$ $y_\alpha^* = 44.85.$

Step 5: The sensitivity of each constraint to the design variable distribution parameters are determined using the MPPs.

Constraint	*Var.*	$\partial P(F)/\partial\mu$	$\partial P(F)/\partial\sigma$
2nd Bend, 250	t	35.6	1.48
	c	17.8	113
	α	32.1	1443
2nd Bend, 500	t	–0.712	–0.031
	c	–0.279	–1.42
	α	–0.608	–26.77
Tip displ.	t	8.31	0.357
	c	3.72	22.0
	α	7.21	323

Step 6: The cost sensitivities are based on manufacturing data. The cost sensitivities are normalized representing the cost of changing the design variable distribution parameters one unit. The cost to shift the nominal value is represented by $\partial\text{Cost}/\partial\mu$. The cost to change the standard deviation (manufacturing tolerance) is represented by $\partial\text{Cost}/\partial\sigma$. Note that there is no cost associated with shifting the nominal compliance and ply angle. Also, note that the sensitivities to changing the standard deviation are negative. This is because a reduction in the standard deviation requires a tightening of tolerance and this usually requires an increase in cost.

Variable	$\partial\text{Cost}/\partial\mu$	$\partial\text{Cost}/\partial\sigma$
t	14.06	–6.81
c	0.0	–12.27
α	0.0	–22.123.

Step 7: The optimization model is now formulated. The objective is to minimize the cost of the airfoil. The constraints are to ensure that the probability of the vibratory frequencies interfering with the excitation frequencies is less then 10% and the probability of the tip displacement being greater than 0.4 inches is less than 20%. (Only the constraints for the second bending mode and tip displacement are shown for brevity.)

$$\begin{aligned}\text{Min: } & 14.06\Delta\mu_t + 0.0\Delta\mu_c + 0.0\Delta\mu_\alpha \\ & - 6.81\Delta\sigma_t - 12.27\Delta\sigma_c - 22.135\Delta\sigma_\alpha.\end{aligned} \tag{14}$$

Subject to

$$\begin{aligned}
0.187 - 0.10 &\le 35.6\Delta\mu_t + 17.8\Delta\mu_c + 32.1\Delta\mu_\alpha + 1.48\Delta\sigma_t + 113\Delta\sigma_c + 1443\Delta\sigma_\alpha \\
0.496 - 0.10 &\le -0.712\Delta\mu_t - 0.279\Delta\mu_c - 0.608\Delta\mu_\alpha - 0.031\Delta\sigma_t - 1.42\Delta\sigma_c - 26.77\Delta\sigma_\alpha \\
0.01 - 0.20 &\le 8.31\Delta\mu_t + 3.72\Delta\mu_c + 7.21\Delta\mu_\alpha + 0.357\Delta\sigma_t + 22.0\Delta\sigma_c + 323\Delta\sigma_\alpha \\
\mu_t &> 0.0316 \\
\mu_t &< 0.1416 \\
&\vdots \\
\sigma_\alpha &> 1.0 \\
\sigma_\alpha &< 12.5.
\end{aligned}$$

Step 8: The optimization problem is solved, reducing the airfoil cost by 123.9 units. The changes in the design parameters are as follows:

Parameter	*Change*	*New Value*
μ-Thickness	–0.010	0.032
μ-Compliance	–1.375	5.000
μ-Ply Angle	–29.0	16.0
σ-Thickness	0.006	0.010
σ-Compliance	–0.275	1.000
σ-Ply Angle	0.68	5.68

The reliability of the new design is re-evaluated as in by returning to Step 4. The probability of no frequency interference ($<10\%$) is not met for the following modes:

- 1st Torsion Mode at 225–275 Hz.
- 2nd Bending mode at 225–275 Hz.
- 3rd Bending mode at 450–550 Hz.

The optimization model was reformulated using the new probabilities of failure and MPP from the new Step 4. The resulting design parameters for the second iteration are:

Parameter	*Change*	*New Value*
μ-Thickness	0.076	0.108
μ-Compliance	0.0	5.0
μ-Ply Angle	3.5	19.5
σ-Thickness	0.0	0.010
σ-Compliance	0.0	1.000
σ-Thickness	–0.229	5.425

The reliability of the new design is assured by returning to Step 4.

The airfoil cost was increased by 19.2 over the first iteration resulting in a 104.7 reduction in cost over the initial design (while meeting POF requirements). The example of an airfoil demonstrated optimization through consideration of both reliability and cost. It was established that the final design configuration have a probability of frequency interference no greater than 10 percent and the initial (example) design configuration was determined to have a probability of frequency interference of 18.7 percent. The proposed uncertainty-based optimization method recommended a final design that met the POF criteria and produced a cost savings.

Employing the reliability/cost optimization method, the probabilistic cost-reliability model refined the design until the cost was minimized and reliability requirements of the product were met. The most advantageous design changes available at each stage of the design refinement process were determined. By continually moving in the direction of design improvement, and by considering reliability, performance and cost, the method represented a multidisciplinary design optimization method.

5 Conclusion

This paper demonstrated that an optimization framework can be integrated with design analysis and probability methods to optimize a design for reliability and cost. The methodology considers product performance, reliability and cost. This allows the engineer to evaluate the impact of design changes on all relevant measures of the design simultaneously. By considering variation in the design parameters, the influence of uncertainty is determined and the design is optimized with sufficient reliability while minimizing manufacturing cost.

Acknowledgment

The research was sponsored by NASA Langley Research Center, Langley, VA.

References

Anonymous. (1980) *Machining Data Handbook*, Cincinnati, OH: Machinability Data Center.

Kowal, M. T., Dey, A. and Tryon, R. G. (1998) 'Integrated design method for probabilistic design', *Annual Reliability and Maintainability Symposium*, Jan 19–22 1998, Anaheim, CA.

Kwak, B. M. and Lee, T. W. (1987) 'Sensitivity analysis for reliability-based optimization using an AFOSM method', *Computer & Structures*, 27, 399–406.

Madsen, H. O., Krenk, S. and Lind, N. C. (1986) *Methods of Structural Safety*, Englewood Cliffs, NJ: Prentice-Hall.

Pu, Y., Das, P. K. and Faulkner, D. (1997) 'A strategy for reliability-based optimization', *Engineering Structures*, 19, 276–282.

Rackwitz, R. and Fiessler, B. (1987) 'Structural reliability under combined random load sequences', *Computer & Structures*, 9, 489–494.

Reddy, M. V., Grandhi, R. V. and Hopkins, D. A. (1994) 'Reliability based structural optimization: a simplified safety index approach', *Computer & Structures*, 53, 1407–1418.

Rosenblatt, E. (1952) 'Remarks on a multivariate transformation', *Ann. Math. Stat.*, 23, 470–472.

Trucks, H. E. (1987) *Designing for Economical Production*, Dearborn, MI: Society of Manufacturing Engineers.